

Wage Rigidities and Jobless Recoveries

Robert Shimer*

November 1, 2010

Abstract

This paper explores how real wage rigidities can generate jobless recoveries. Suppose that after a transitory shock, the capital stock lies below trend. If wages are flexible, they decline as the economy grows back to trend. If wages are completely rigid and the labor market is otherwise frictionless, the transitory shock causes a proportional and permanent decline in employment, capital, output, consumption, and investment relative to trend. In a search model with rigid wages, the impact of the shock eventually disappears but quantitatively the behavior of the economy is very similar.

*I am grateful for comments from Steven Davis, Rafael Lavive, Ivan Werning, and numerous seminar participants on an earlier version of this paper, and for financial support from the National Science Foundation.

1 Introduction

This paper explores the implications of wage rigidities for the propagation of shocks. Suppose an economy that is on a balanced growth path is hit by a transitory shock. While the shock is “on,” low investment reduces the level of the capital stock below the balanced growth path. This paper is concerned with the subsequent dynamics of the economy, after the shock is turned “off.” If the labor market clears, wages must fall, employment typically rises, and the economy reaccumulates capital and returns to the balanced growth path. If instead the growth rate of real wages is unchanged and employment is determined by labor demand, there is a proportional decline in employment and in capital, output, consumption, and investment relative to trend. The economy exhibits a jobless recovery.

The same logic holds if there are search frictions in the labor market. If wages decentralize the social planner’s problem, they are low during the recovery, employment is typically high, and the economy returns to the balanced growth path. If wages are rigid, there is a long-lasting (but not permanent) decline in employment, capital, output, consumption, and investment relative to trend. Search frictions are also useful for rationalizing why wages continue to grow on trend after an adverse shock. From the perspective of individual workers and firms in the search economy, expectations of trend wage growth is as plausible as expectations of flexible wages, and in particular are consistent with individual rationality.

The empirical motivation for this paper is the anemic behavior of the U.S. economy during the 2000s and the disastrous experience since the onset of the financial crisis in 2008 (Figure 1). From the first quarter of 2008 until the second quarter of 2010—the latest available data at the time this paper was written—the employment-population ratio fell by 7.0 log points and average per capita weekly hours fell by 8.3 log points. Over the same time period, real GDP per capita grew 6.9 log points slower than trend and real nondurable and services consumption per capita grew 6.6 log points slower than trend.¹ This extraordinary contraction came on the heels of a decade of weak growth, with essentially no recovery from the mild recession at the start of the decade. Many forecasters predict similarly weak growth in the years ahead and few see any prospect for an early return to the employment level in 2007 and certainly not to the level in 2000.

¹I construct a linear trend from 1951 to 2008. Over this time period, the employment-population ratio grew by 0.07 log points per quarter and average weekly hours grew by 0.04 log points, so detrending these variables is quantitatively unimportant. On the other hand, GDP per capita and consumption per capita grew by 0.45 and 0.52 log points per quarterly, respectively, so here detrending is more important. The theory I develop is consistent with balanced growth. That is, trend productivity growth causes trend increases in output and consumption without affecting employment or hours, offering a theoretical justification for the differential treatment of these time series. But in practice it is not important whether I detrend employment and hours.

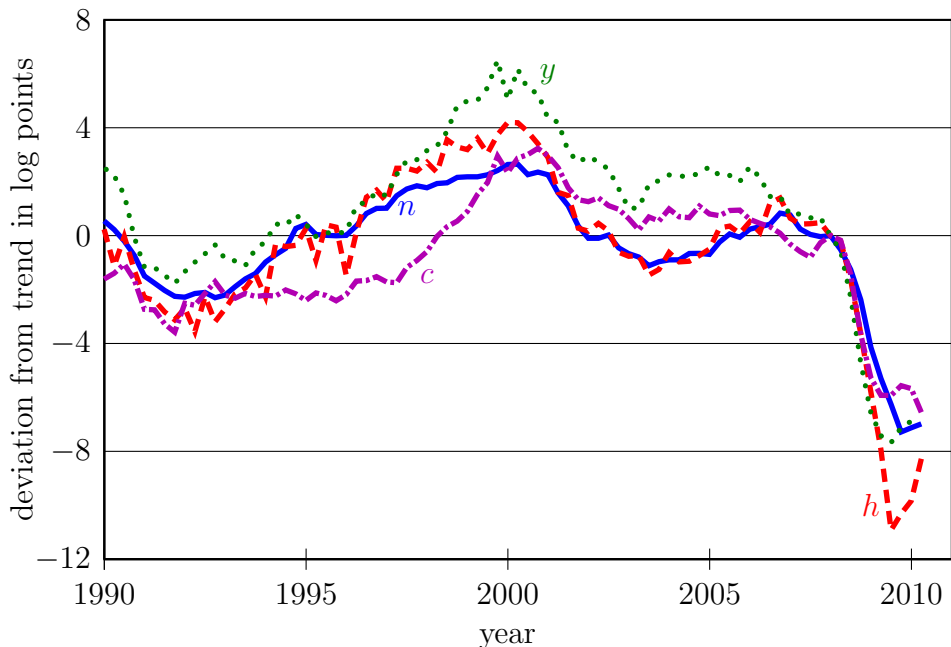


Figure 1: The solid blue line shows employment per capita n ; the dashed red line shows hours per capita h ; the dotted green line shows GDP per capita y ; and the dash-dotted purple line shows consumption per capita c . GDP and consumption are detrended using a linear trend from 1951 to 2008 and expressed as deviations from trend in log points.

Real labor costs have been quite stable throughout this episode. To show this, I use data from the Employment Cost Index for total compensation, deflated by the consumer price index for all urban consumers and detrended.² This is essentially a price index, measuring the change in the total cost of a hiring a worker to fill a specific, narrowly defined position. Figure 2 shows little evidence for a decrease in employment costs relative to trend during the 2001 recession, although they did start to fall several years later as the real economy reached its trough. Employment costs started rising again in the middle of 2008, mirroring a decrease in price inflation at this stage of the recession. By the second quarter of 2008, employment costs were essentially back at the same level as in 2007. The model predicts a close link between employment costs and productivity and the figure also confirms this. Both output per worker nor output per hour are back at their 2007 levels as well, with time patterns that closely mirror the path of labor costs. In short, Figures 1 and 2 do not provide any evidence that real wages are procyclical.

The basic building blocks of this paper are not new. The underlying framework is a neoclassical model, where firms use capital and labor to produce output and individuals

²The trend for employment costs is 0.17 log points per quarter. The trend for output per hour and output per worker is 0.41 and 0.38 log points.

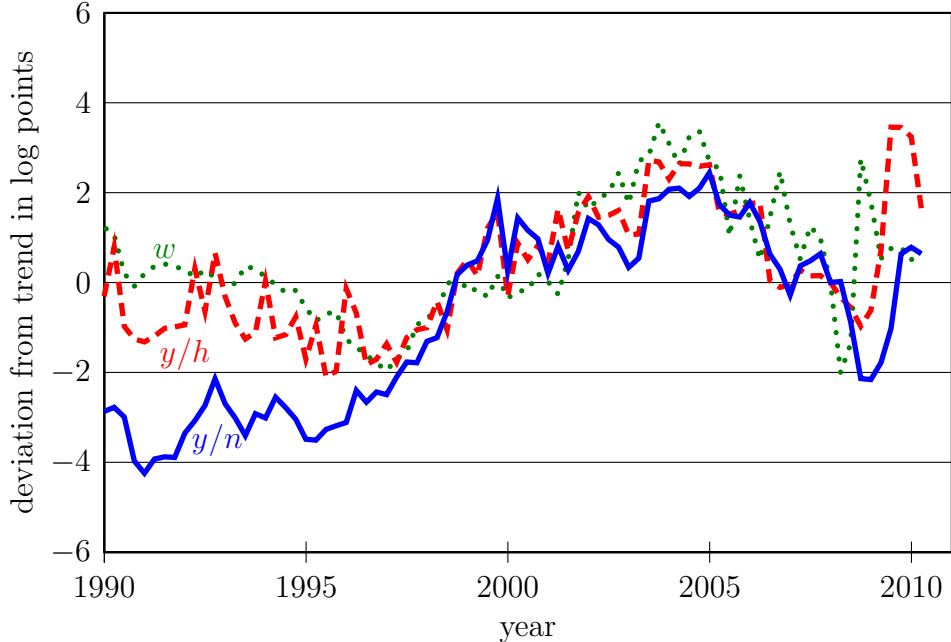


Figure 2: The solid blue line shows output per capita y/n ; the dashed red line shows output per hour y/h ; and the dotted green line shows employment costs w . All three variables are detrended using a linear trend from 1951 to 2008 and expressed as deviations from trend in log points.

supply labor and save to maximize their utility. I first introduce wage rigidities directly into this model. I make the extreme assumption that wages grow at the same rate as labor augmenting technology and workers are required to supply whatever labor firms demand. Any difference between labor supply and employment is naturally labeled as unemployment.

While this first model is useful for understanding the consequences of wage rigidities, it is not so useful for understanding why rigid wages may prevail in equilibrium. In particular, this model cannot explain why unemployed workers could not offer to work at less than the prevailing wage (Barro, 1977). Here search frictions are useful. Since Diamond (1971), economists have understood that a small amount of search frictions can lead to an equilibrium that is far from the competitive one. While each worker-firm pair must pay a wage that is close to the wage paid in all other matches, this common wage may potentially take on quite different values, depending on expectations. In particular, there is an interval of wages at which firms are willing to employ workers and workers are willing to work, and this interval may be quite large even if search frictions are relatively unimportant (Hall, 2005). To be clear, however, the model does not explain why wages are rigid, since the flexible wage path is also consistent with individual rationality. Instead, it makes the point that conditional on a path of individually rational wages, the model is consistent with jobless recoveries if and only if wages are rigid.

The literature subsequent to Hall (2005) has focused on whether wage rigidities help to amplify shocks. To my knowledge, this is the first paper emphasizing how wage rigidities can generate jobless recoveries. Moreover, the previous literature has focused on the response to a particular (aggregate technology) shock. This paper stresses that the behavior of the model does not depend so much on the nature of the shock, but rather on the amount that the capital stock declines and on the subsequent adjustment of wages.

I start my analysis in a neoclassical growth model with a competitive labor market in which supply and demand are always equal. I consider the transitional dynamics when the capital stock starts off below trend. I show that during the subsequent adjustment dynamics, the real wage always falls below trend. Depending on preferences, employment may rise (because individuals are poorer) or fall (because the wage is lower). For example, if the period utility function can be expressed as log consumption minus the disutility of work, employment and investment rise on the impact of an adverse shock, while consumption and output fall. In any case, the model never delivers a quantitatively large decline in employment in response to an adverse reallocation shock.

I then consider an ad hoc wage rigidity in the neoclassical model. I assume that the wage is fixed at the balanced growth path of the neoclassical model and the equilibrium level of employment is determined from firms' labor demand decision, without regard to workers' willingness to supply this labor. I prove that the level of output, employment, consumption, and investment is proportional to the capital stock. That is, if a one-time shock destroys one percent of the capital stock, other economic outcomes will also fall by one percent and never recover. This part of the paper is a warmup to the main exercise, a model with search frictions, but it is useful because I can obtain many results in closed form and the same forces are at work in both models.

I next show how this logic carries over to a model with search frictions. I augment the neoclassical growth model with a recruiting technology through which a firm uses some of its employees to attract unemployed workers (Shimer, 2010). As Rogerson and Shimer (forthcoming) discuss, the natural analog of the (flexible wage) neoclassical growth model is the social planner's solution to that model. I confirm that the main insights of the frictionless model carry over to the search model. For example, if the period utility function can be expressed as log consumption minus the disutility of work, employment and investment increase following an adverse shock, while consumption and output decline. With strong complementarities between consumption and labor supply, it is possible to generate a decline in all four outcomes; however, employment is still much less volatile than output.

Finally, I introduce rigid wages into the search model. I assume that the wage path is fixed at the level that decentralizes the balanced growth path in the social planner's problem.

Unlike in the frictionless model, I do not impose that workers have to supply whatever labor firms demand at that wage. Instead, I prove that in equilibrium firms are always willing to employ workers at this fixed wage, although they cut back on recruiting if the wage is too high. Workers are also always willing to work at the wage and indeed would happily take a job at a lower wage if one were available. Thus this wage is consistent with the indeterminacy in equilibrium wages identified by Hall (2005). In this case I verify that, even if consumption and leisure are separable, the model can a decline in output, consumption, investment, and employment relative to trend, with all four detrended variables moving by roughly the same magnitude. The model is therefore broadly consistent with the empirical patterns that we have observed during the last two years.

The model predicts that employment is low during a recovery because firms cut back on hiring, not because the incidence of unemployment rises. I have argued elsewhere that this view accounts for the majority of fluctuations in unemployment (Shimer, 2007), a view that was reaffirmed during the current recovery (Elsby, Hobijn, and Şahin, forthcoming). In particular, this approach accounts for the simultaneous increase in unemployment and decline in vacancies that occurs during most recessions. On the other hand, this approach misses out on some features of the current recession, especially the increase in job vacancies during a period of constant unemployment in the second half of 2009 and first half of 2010. I leave this breakdown in the Beveridge curve as a topic for future research.

The paper has three more sections. The next section analyzes the frictionless model. I focus on a deterministic environment for simplicity and cover first the planner's solution, second the decentralized equilibrium, and third the rigid wage model. This develops the intuition for the following section, which analyzes the model with search frictions. It again contains three subsections which go through the same versions of the model: planner's solution, decentralization, and rigid wage model. The final section concludes with a further discussion of the interpretation of reallocation shocks, the implications of these shocks for measured total factor productivity, and the consequences of wage rigidities for countercyclical fiscal policy.

2 Frictionless Model

I consider an economy with a representative household and a representative firm. The household contains a unit measure of individuals with identical preferences. Household members are infinitely-lived and discount the future with factor $\beta \in (0, 1)$. Labor is indivisible and in period t a fraction n_t of household members are employed. The household maximizes the equal-weighted sum of its members' utility, but the marginal utility of consumption depends

on employment status and so in general employed and unemployed household members will not consume the same amount.³ In particular, the period utility of an employed household member who consumes c_e is

$$\frac{c_e^{1-\sigma}(1 + (\sigma - 1)\gamma)^\sigma}{1 - \sigma}$$

if $\sigma \neq 1$ and $\log c_e - \gamma$ if $\sigma = 1$; the corresponding value for an unemployed household member who consumes c_u is

$$\frac{c_u^{1-\sigma}}{1 - \sigma}$$

if $\sigma \neq 1$ and $\log c_u$ if $\sigma = 1$. This specification of preferences is consistent with balanced growth: a proportional change in labor and non-labor income leaves the household's choice of employment unchanged and simply leads to a proportional change in consumption. The parameter $\gamma > 0$ determines the disutility of work. The parameter $\sigma > 0$ plays several roles. It determines risk-aversion and the intertemporal elasticity of substitution. It is also important for the complementarity between consumption and work and in particular determines the relative consumption of employed and unemployed household members. As I show below, if $\sigma > 1$, employed workers optimally consume more than unemployed workers.

Firms use capital k and labor n to produce output with a standard Cobb-Douglas production technology. The output $A_t^{1-\alpha}k^\alpha n^{1-\alpha}$ plus undepreciated capital $(1 - \delta)k$ is then used both for investment and consumption. I assume labor augmenting technology grows at a constant rate $g \geq 0$, $A_{t+1} = (1 + g)A_t$. The parameter $\alpha \in (0, 1)$ is the capital share of income.

In what follows, I first solve a benevolent planner's problem, then I discuss the (standard) decentralization as a competitive economy. This is essentially the Hansen (1985) model. Finally I develop the rigid wage model. Throughout I assume parameter values are such that some household members are employed and some are unemployed.

2.1 Planner's Problem

A planner chooses the time path of consumption for employed workers $\{c_{e,t}\}$, consumption for unemployed workers $\{c_{u,t}\}$, employment $\{n_t\}$, and the capital stock $\{k_{t+1}\}$ to maximize the total utility of the representative household

$$\sum_{t=0}^{\infty} \beta^t \left(n_t \frac{c_{e,t}^{1-\sigma}(1 + (\sigma - 1)\gamma)^\sigma}{1 - \sigma} + (1 - n_t) \frac{c_{u,t}^{1-\sigma}}{1 - \sigma} \right)$$

³In the frictionless model with a competitive labor market, it would be more correct to refer to household members as nonemployed rather than unemployed, since the activity is in no sense involuntary. I use this notation for consistency with the remainder of the paper.

subject to a law of motion for the capital stock

$$k_{t+1} = A_t^{1-\alpha} k_t^\alpha n_t^{1-\alpha} + (1 - \delta)k_t - n_t c_{e,t} - (1 - n_t)c_{u,t}, \quad (1)$$

taking as given the initial capital stock k_0 .

To solve the model, place a multiplier $\beta^t \lambda_t$ on the law of motion for capital. I obtain three sets of conditions describing a solution. The first one gives the allocation of consumption between employed and unemployed workers as

$$c_{e,t} = \frac{c_t(1 + (\sigma - 1)\gamma)}{1 + (\sigma - 1)\gamma n_t} \text{ and } c_{u,t} = \frac{c_t}{1 + (\sigma - 1)\gamma n_t}, \quad (2)$$

where $c_t = n_t c_{e,t} + (1 - n_t)c_{u,t}$ is total consumption. In particular, the ratio of consumption by the employed to consumption by the unemployed is $c_{e,t}/c_{u,t} = 1 + (\sigma - 1)\gamma$. If $\sigma > 1$, the employed consume more than the unemployed because working raises the marginal utility of consumption. This is why I refer to σ as the complementarity between consumption and employment. In addition,

$$\lambda_t = \left(\frac{c_t}{1 + (\sigma - 1)\gamma n_t} \right)^{-\sigma}, \quad (3)$$

so λ_t is the marginal utility of consumption of a hypothetical representative agent with period utility $c_t^{1-\sigma}(1 + (\sigma - 1)\gamma n_t)^\sigma / (1 - \sigma)$ when he consumes c_t and works a fraction n_t of his time.

The second condition is the Euler equation,

$$\lambda_t = \beta \lambda_{t+1} (\alpha \kappa_{t+1}^{\alpha-1} + 1 - \delta), \quad (4)$$

where $\kappa_t \equiv k_t / A_t n_t$ is capital per efficiency unit of labor. This states that the planner must be indifferent between having households consume one additional unit at t , valued at the marginal utility of consumption λ_t , or invest it, with marginal product of capital $\alpha \kappa_{t+1}^{\alpha-1} + 1 - \delta$ at $t + 1$, valued at the discounted marginal utility of consumption at $t + 1$, $\beta \lambda_{t+1}$. These first two conditions hold in every model in this paper.

The final condition is the optimality of employment. I assume throughout that some household members work and some are unemployed; this will be the case if the disutility of labor is sufficiently large. At an interior solution for employment, the first order condition for labor is

$$\gamma \sigma \lambda_t^{-\frac{1}{\sigma}} = (1 - \alpha) A_t \kappa_t^\alpha. \quad (5)$$

This equates the marginal rate of substitution between consumption and leisure to the

marginal product of labor.

It is straightforward to solve for a balanced growth path along which consumption of employed and unemployed workers, capital, and output grow at rate g , the marginal utility of consumption grows at rate $(1+g)^{-\sigma} - 1$, and employment is constant. But I am primarily interested in how the economy behaves when the capital stock is initially below the balanced growth path. To answer this, I use equation (5) to eliminate κ_{t+1} from equation (4) and equations (2), (3), and (5) to eliminate n_t , $c_{e,t}$, and $c_{u,t}$ from equation (1). This leaves

$$\lambda_t = \beta \lambda_{t+1} \left(\alpha \left(\frac{(1-\alpha)A_{t+1}\lambda_{t+1}^{1/\sigma}}{\gamma\sigma} \right)^{\frac{1-\alpha}{\alpha}} + 1 - \delta \right),$$

$$k_{t+1} = \left(\left(\frac{(1-\alpha)A_t\lambda_t^{1/\sigma}}{\gamma\sigma} \right)^{\frac{1-\alpha}{\alpha}} \frac{1-\alpha+\alpha\sigma}{\sigma} + 1 - \delta \right) k_t - \lambda_t^{-\frac{1}{\sigma}}.$$

In particular, I log-linearize it in a neighborhood of the balanced growth path. Using $\hat{\lambda}_t$ to denote the log-deviation of λ_t from the balanced growth path, and similarly for \hat{k}_t , I obtain

$$\hat{\lambda}_{t+1} = \varepsilon_\lambda \hat{\lambda}_t,$$

$$\hat{k}_{t+1} = \left(\frac{\varepsilon_k - \alpha - \frac{(1-\alpha)(1-\delta)}{1+g}}{\alpha\sigma} \right) \hat{\lambda}_t + \varepsilon_k \hat{k}_t,$$

where

$$\varepsilon_\lambda = \frac{\alpha\sigma}{1 - \alpha + \alpha\sigma - \beta(1+g)^{-\sigma}(1-\alpha)(1-\delta)} \text{ and } \varepsilon_k = \frac{(1+g)^{\sigma-1}}{\beta\varepsilon_\lambda}.$$

One can verify that the system is saddle path stable, with eigenvalues ε_λ and ε_k satisfying $0 < \varepsilon_\lambda < 1 < \varepsilon_k$. The eigenvector associated with the stable arm is $(\hat{k}, \hat{\lambda}) = (1, s)$, where

$$s \equiv -\frac{\alpha\sigma(\varepsilon_k - \varepsilon_\lambda)}{\varepsilon_k - \alpha - \frac{(1-\alpha)(1-\delta)}{1+g}}$$

is the elasticity of the marginal utility of consumption λ with respect to the capital stock along the saddle path. The slope s is negative, so when the capital stock is below trend, the marginal utility of consumption is above trend.

Now suppose that after some unmodeled adverse shock, the economy starts with one percent less capital than in the balanced growth path, $\hat{k}_0 = -1$. Following this event, the economy stays on the saddle path, and so $\hat{\lambda}_0 = -s$, above its trend value. Eventually $\hat{\lambda}$ falls back to trend as the capital stock is rebuilt. I am interested in how other model outcomes behave along this transition path. The discussion that follows focuses on the log-linear dynamics in a neighborhood of the balanced growth path.

To start, equation (5) implies labor productivity $A_t \kappa_t^\alpha$ moves inversely with λ_t ,

$$A_t \kappa_t^\alpha \propto \lambda_t^{-\frac{1}{\sigma}}.$$

A one log point decrease in the capital stock raises λ by $-s$ log points; therefore, labor productivity must fall upon the impact of the shock, by $-s/\sigma$ log points. It then gradually increases back to its trend value.

Next consider the impact on employment. Equation (5) implies

$$\begin{aligned} \hat{n}_0 &= \hat{k}_0 + \frac{1}{\alpha\sigma} \hat{\lambda}_0 = -\frac{\alpha\sigma + s}{\alpha\sigma} \\ &= -\frac{\varepsilon_\lambda - \alpha - (1+g)(1-\alpha)(1-\delta)}{\varepsilon_k - \alpha - (1+g)(1-\alpha)(1-\delta)} \end{aligned}$$

If $\sigma \leq 1$, one can verify that this is negative, so employment rises on the impact of the adverse shock and then gradually falls back to its trend value. This seems like the most natural case. Households are poor and since leisure is a normal good, they cut back on it and so supply more labor. This accelerates the economy's recovery to trend. But this need not be the case. For example, suppose $g = 0$ so there is no growth. In this case, if

$$\sigma > \frac{(1 - \beta(1 - \delta))(1 - \delta(1 - \alpha))}{\alpha\delta} \equiv \bar{\sigma},$$

employment initially falls after the adverse shock before rising back to its trend value because substitution effects dominate income effects. Following the loss of capital, the marginal product of labor is low, which discourages households from supplying much labor despite their relative poverty. But while this is a theoretical possibility, when I later calibrate the model I find that for any reasonable parameterization, employment rises following the impact of the shock.

One can also compute the behavior of consumption, although the expressions are messier. If $\sigma \leq 1$, it is possible to prove analytically that when the capital stock is below trend, consumption is also below trend. If there is no growth, $g = 0$, this is also true when $\sigma \geq \bar{\sigma}$, the case where a reduction in the capital stock lowers employment, although the result may be reversed at intermediate values.

Finally, I turn to investment, $i = Ak^\alpha n^{1-\alpha} - c$. In general, the elasticity of investment with respect to the capital stock is also a messy expression, but I find numerically that when $\sigma \leq 1$, a reduction in the capital stock raises investment as the economy returns to trend. This can again be reversed if σ is sufficiently large. In particular, when $g = 0$ and $\sigma = \bar{\sigma}$, a one percent reduction in the capital stock reduces investment by α percent. Note, however,

that the decline in investment is always smaller than the decline in the capital stock and so capital does indeed return to the initial trend.

In summary, at low values of σ , an economy with less capital than trend has high employment and consumption. But for large σ , it is at least theoretically possible to have employment, consumption, and investment all below their trend values, although employment always falls by less than does the capital stock, pushing down labor productivity.

2.2 Decentralized Equilibrium

The welfare theorems apply, and so a competitive equilibrium decentralizes the social planner's problem. This feature of the neoclassical growth model is well understood, but I write down the decentralized economy formally as a prelude to the discussion of a model in which wages do not clear the labor market. I assume throughout this paper that markets are complete. Rather than specify trade in securities that insure against any aggregate risk, I assume that insurance takes place within a large household (Merz, 1995).

A typical household with initial assets a_0 chooses a path for consumption $\{c_{e,t}, c_{u,t}\}$ and employment to maximize the average utility of its members,

$$\sum_{t=0}^{\infty} \beta^t \left(n_t \frac{c_{e,t}^{1-\sigma} (1 + (\sigma - 1)\gamma)^\sigma}{1 - \sigma} + (1 - n_t) \frac{c_{u,t}^{1-\sigma}}{1 - \sigma} \right),$$

subject to a lifetime budget constraint

$$\sum_{t=0}^{\infty} q_0^t (n_t c_{e,t} + (1 - n_t) c_{u,t} - w_t n_t) = a_0,$$

where q_0^t is the time 0 price of a unit of consumption at time t and w_t is the wage. As in the social planner's problem, the household equates the marginal utility of consumption for employed and unemployed individuals, giving equation (2). In addition, the household problem yields the Euler equation

$$\frac{q_0^{t+1}}{q_0^t} = \beta \frac{\lambda_{t+1}}{\lambda_t}, \tag{6}$$

where λ_t is the marginal utility of consumption for a hypothetical representative agent, defined in equation (3). Finally, the household equates the marginal rate of substitution for the representative agent to the wage,

$$\gamma \sigma \lambda_t^{-\frac{1}{\sigma}} = w_t.$$

Next, a firm that initially owns k_0 units of capital chooses a path for capital and employment to maximize the present value of its profits,

$$\sum_{t=0}^{\infty} q_0^t (A_t^{1-\alpha} k_t^\alpha n_t^{1-\alpha} + (1-\delta)k_t - k_{t+1} - w_t n_t).$$

The first order condition for capital is

$$q_0^t = q_0^{t+1} (\alpha \kappa_{t+1}^{\alpha-1} + 1 - \delta).$$

Eliminating q_0^{t+1}/q_0^t using the household Euler equation (6) gives the planner's Euler equation (4). In addition, the firm's first order condition for labor equates the marginal product of labor to the wage,

$$(1-\alpha)A_t \kappa_t^\alpha = w_t. \quad (7)$$

Combining this with the household's labor supply decision recovers the planner's first order condition (5). This establishes the equivalence between the planner's problem and the decentralized equilibrium.

One important prediction that comes out of the decentralization concerns the behavior of wages. Recall that when the economy has too little capital, labor productivity is below the balanced growth path. With a Cobb-Douglas production function, the wage is proportional to labor productivity and so the wage falls as well. I now turn to a model in which the wage cannot adjust.

2.3 Rigid Wage

In this section, I assume that the wage grows at the rate of technological progress, $w_t = (1+g)^t w_0$ for some w_0 , and households must supply whatever labor firms demand at that wage. The model is otherwise unchanged. In particular, a household with initial assets a_0 chooses a path for consumption $\{c_{e,t}, c_{u,t}\}$ to maximize the average utility of its members,

$$\sum_{t=0}^{\infty} \beta^t \left(n_t \frac{c_{e,t}^{1-\sigma} (1 + (\sigma-1)\gamma)^\sigma}{1-\sigma} + (1-n_t) \frac{c_{u,t}^{1-\sigma}}{1-\sigma} \right),$$

subject to a lifetime budget constraint

$$\sum_{t=0}^{\infty} q_0^t (n_t c_{e,t} + (1-n_t) c_{u,t} - w_t n_t) = a_0,$$

taking as given a path for intertemporal prices, the wage, and the level of employment. The household Euler equation (6) is unchanged. This, combined with the household budget constraint, completely describes the household's behavior. In particular, the household does not choose how much to work.

The firm's problem is unchanged, since the firm simply maximizes profits taking as given an arbitrary sequence for wages and intertemporal prices. Combining the first order condition for capital with the household Euler equation, eliminating employment using equation (7), and using the assumption that $w_t/A_t = w_0/A_0$, I get

$$\lambda_t = \beta \left(\alpha \left(\frac{(1-\alpha)A_0}{w_0} \right)^{\frac{1-\alpha}{\alpha}} + 1 - \delta \right) \lambda_{t+1} = \beta R_0 \lambda_{t+1}, \quad (8)$$

where

$$R_0 \equiv \alpha \left(\frac{(1-\alpha)A_0}{w_0} \right)^{\frac{1-\alpha}{\alpha}} + 1 - \delta$$

is the gross marginal product of capital. In addition, use equations (3) and (7) to eliminate consumption and employment from the resource constraint:

$$k_{t+1} = \left(\left(\frac{(1-\alpha)A_0}{w_0} \right)^{\frac{1-\alpha}{\alpha}} + 1 - \delta \right) k_t - \lambda_t^{-\frac{1}{\sigma}} \left(1 + (\sigma - 1)\gamma \left(\frac{(1-\alpha)A_0}{w_0} \right)^{\frac{1}{\alpha}} \frac{k_t}{A_t} \right). \quad (9)$$

This is again a pair of difference equations for capital and the marginal utility of consumption.

Now suppose that the economy starts from a balanced growth path where $\beta R = (1 + g)^\sigma$. This might, for example, be the decentralized equilibrium of an economy with a competitive labor market that is moving along a balanced growth path. There is a one time, unanticipated, one percent decrease in the capital stock. Thereafter, there are no more shocks but the wage is forced to grow at rate g . I claim that there is a unique equilibrium of the economy in which employment, consumption, investment, and output all fall by one percent on the impact of the shock and thereafter grow at their usual growth rate: g for consumption, investment, and output, with employment constant at the depressed level. This is a jobless recovery.

To prove this, first observe from equation (8) that if $\beta R = (1 + g)^\sigma$, $\lambda_{t+1} = (1 + g)^{-\sigma} \lambda_t$ along the transition path. Then rewrite (9) as

$$k_{t+1} = \left(\left(\frac{(1-\alpha)A_0}{w_0} \right)^{\frac{1-\alpha}{\alpha}} + 1 - \delta - A_0^{-1} \lambda_0^{-\frac{1}{\sigma}} (\sigma - 1)\gamma \left(\frac{(1-\alpha)A_0}{w_0} \right)^{\frac{1}{\alpha}} \right) k_t - \lambda_0^{-1/\sigma} (1 + g)^t.$$

Now fix

$$\lambda_0^{-\frac{1}{\sigma}} = \frac{\left(\left(\frac{(1-\alpha)A_0}{w_0} \right)^{\frac{1-\alpha}{\alpha}} - \delta - g \right) k_0}{1 + A_0^{-1}(\sigma - 1)\gamma \left(\frac{(1-\alpha)A_0}{w_0} \right)^{\frac{1}{\alpha}} k_0}.$$

Substituting into the difference equation gives $k_{t+1} = (1+g)k_t$, consistent with the proposed equilibrium. Moreover, at any higher value of $\lambda_0^{-\frac{1}{\sigma}}$, the economy runs out of capital in finite time. At any lower value, the transversality condition is violated. Therefore this value of λ_0 is the only possible equilibrium. Along the subsequent balanced growth path, capital grows at rate g , the marginal utility of consumption shrinks at rate $(1+g)^{-1/\sigma} - 1$, and employment is constant. This implies that consumption and investment grow at rate g as well.

To summarize, with rigid wages, the model naturally produces a permanent and equal decline in consumption, employment, investment, and output relative to trend in response to a one time shock that reduces the size of the capital stock. The question that cannot be answered in the frictionless model is why wages would not adjust following the shock. There are unemployed workers who would like to work at less than the prevailing wage and firms that would be willing to hire them. What prevents these workers from offering to work at that wage? Search frictions, which I introduce next, provide a natural answer.

3 Search Model

I now extend the neoclassical growth model by assuming that firms have access to two technologies. One uses capital and labor to produce output. The other uses labor to recruit more workers. Each period, every firm divides its workforce between these two tasks, production and recruiting. This allocation determines current output and the evolution of the aggregate employment rate, which is now a state variable. I again start by looking at the planner's solution and then discuss its decentralization before turning to the economy with a rigid wage.

3.1 Planner's Problem

A planner chooses the time path of consumption $\{c_{e,t}, c_{u,t}\}$, recruiting $\{v_t\}$, employment $\{n_{t+1}\}$, and the capital stock $\{k_{t+1}\}$ to maximize the total utility of the representative household

$$\sum_{t=0}^{\infty} \beta^t \left(n_t \frac{c_{e,t}^{1-\sigma} (1 + (\sigma - 1)\gamma)^\sigma}{1 - \sigma} + (1 - n_t) \frac{c_{u,t}^{1-\sigma}}{1 - \sigma} \right)$$

subject to two constraints. First, the economy faces a resource constraint

$$k_{t+1} = A_t^{1-\alpha} k_t^\alpha (n_t - v_t)^{1-\alpha} + (1 - \delta)k_t - n_t c_{e,t} - (1 - n_t)c_{u,t}. \quad (10)$$

The only modification in this constraint, compared to the frictionless model, is in production. Of the n_t employed workers, v_t are dedicated to recruiting and $n_t - v_t$ to production, reducing the available output. The recruiters are used to attract new workers to the firm, with their success determined by a matching function. In particular, employment evolves as

$$n_{t+1} = (1 - x)n_t + m(v_t, 1 - n_t), \quad (11)$$

where x is the constant fraction of employed workers who lose their job and $m(v_t, 1 - n_t)$ is the number of new matches, an increasing function, constant returns to scale function of recruiters and unemployment. The matching function follows Pissarides (1985), although its arguments are recruiters and unemployment rather than vacancies and unemployment. The planner takes as given an initial capital stock k_0 and level of employment n_0 .

To solve this model, I place a multiplier $\beta^t \lambda_t$ on the law of motion for capital and $\beta^t \xi_t$ on the law of motion for employment. The first order conditions for consumption of the employed and unemployed deliver equations (2) and (3). The first order condition for capital delivers the Euler equation (4), with

$$\kappa_t \equiv \frac{k_t}{A_t(n_t - v_t)}$$

redefined as capital per efficiency units of producers. These results are otherwise unchanged from the model without search frictions.

Turn next to the first order condition for recruiting,

$$\lambda_t(1 - \alpha)A_t \kappa_t^\alpha = \xi_t m_v(v_t, 1 - n_t),$$

and employment,

$$\xi_t = \beta \left(\frac{c_{e,t+1}^{1-\sigma} (1 + (\sigma - 1)\gamma)^\sigma}{1 - \sigma} - \frac{c_{u,t+1}^{1-\sigma}}{1 - \sigma} + \lambda_{t+1} \left((1 - \alpha)A_{t+1} \kappa_{t+1}^\alpha - c_{e,t+1} + c_{u,t+1} \right) + \xi_{t+1} (1 - x - m_u(v_{t+1}, 1 - n_{t+1})) \right),$$

where m_v and m_u are the derivatives of the matching function with respect to the number of recruiters and unemployed workers, respectively. Eliminate ξ_t from the second equation using

the first. Also replace c_e and c_u with the marginal utility of consumption using equations (2) and (3). This gives

$$\lambda_t(1 - \alpha)A_t\kappa_t^\alpha = \beta m_v(v_t, 1 - n_t) \left(-\gamma\sigma\lambda_{t+1}^{\frac{\sigma-1}{\sigma}} + \lambda_{t+1}(1 - \alpha)A_{t+1}\kappa_{t+1}^\alpha \left(1 + \frac{1 - x - m_u(v_{t+1}, 1 - n_{t+1})}{m_v(v_{t+1}, 1 - n_{t+1})} \right) \right). \quad (12)$$

This expression determines the optimal level of recruiting versus production. An additional producer raises output at t by the marginal product of labor $(1 - \alpha)A_t\kappa_t^\alpha$. This is valued at the marginal utility of consumption, and so the left hand side is the utility cost of recruiting. Each additional recruiter attracts m_v additional workers to the firm next period. This reduces utility directly by the marginal disutility of working. In addition, it allows the planner to shift resources into production next period, while keeping employment at a fixed level two periods hence. More precisely, given the level of employment next period n_{t+1} , the planner sets the number of producers $l_{t+1} \equiv n_{t+1} - v_{t+1}$ to achieve a desired employment level in two periods, $n_{t+2} = (1 - x)n_{t+1} + m(v_{t+1}, 1 - n_{t+1})$. Implicitly differentiating, I obtain

$$\frac{\partial l_{t+1}}{\partial n_{t+1}} = 1 + \frac{1 - x - m_u(v_{t+1}, 1 - n_{t+1})}{m_v(v_{t+1}, 1 - n_{t+1})}$$

when n_{t+2} is held fixed. These are the additional workers available to produce next period for each worker who is hired this period, and the output is valued at next period's marginal utility of consumption.

In summary, the model has two endogenous state variables, n and k , two controls, c and v , with κ defined for convenience as the capital-producer ratio and λ as the marginal utility of consumption. The four equations (4), (10), (11), and (12) describe the equilibrium. The consumption of employed and unemployed workers can then be recovered from equation (2).

I solve the model numerically. More precisely, I calibrate the model parameters and log-linearize it in a neighborhood of the balanced growth path. I then look at the behavior of the model economy when it starts off away from the balanced growth path. Since there are two state variables, there are potentially many initial conditions to examine. In practice, however, the economy quickly converges to one-dimensional subset of the (k, n) plane, for reasons that I discuss further below.

I first describe the calibration. I think of a time period as a month and set the discount factor to $\beta = 0.996$, just under five percent annually. I fix $\alpha = 0.33$ to match the capital share of income in the National Income and Product Accounts. I set the growth rate of labor augmenting technology at $g = 0.0018$, or 2.2 percent net growth per year, consistent

with the annual measures of multifactor productivity growth in the private business sector constructed by the Bureau of Labor Statistics.⁴ The level of technology A_0 affects the level of output, capital, and consumption, but is irrelevant for the statistics I report here. I set the monthly depreciation rate δ to target a trend capital to annual output ratio of 3.2, the average capital-output ratio in the United States since 1948.⁵ The exact value of δ depends on the preference parameter σ , but in the benchmark model this procedure implies $\delta = 0.0028$.

I turn next to the parameters that determine flows between employment and unemployment. Shimer (2005) measures the average exit probability from employment to unemployment in the United States at $x = 0.034$ per month, and I stick with that number here. Although there are many estimates of the matching function f in the literature (see the survey by Petrongolo and Pissarides, 2001), most papers assume that firms create job vacancies in order to attract unemployed workers and so estimate matching functions using data on unemployment and vacancies. The technology in this paper is slightly different, with firms using workers to recruit workers. Unfortunately I am unaware of any time series showing the number of workers (or hours of work) devoted to recruiting, and so the choice of f is somewhat arbitrary. Still, following much of the search and matching literature, I focus on an isoelastic function, $m(v, u) = \bar{\mu}v^\eta u^{1-\eta}$, and in particular at the symmetric case, $\eta = 0.5$. I discuss below the importance of this parameter. To pin down the efficiency parameter in the matching function $\bar{\mu}$, I build on evidence in Hagedorn and Manovskii (2008) and Silva and Toledo (2009). Those papers argue that recruiting a worker uses approximately 4 percent of one worker's quarterly wage, i.e., a recruiter can attract approximately 25 new workers in a quarter, or 8.33 in a month. I use this fact and data on the average unemployment rate to determine $\bar{\mu}$. I proceed in several steps. First, from (11), the trend employment rate satisfies

$$n = \frac{f}{x + f},$$

where $f \equiv m(v, u)/u$ is the rate that unemployed workers find jobs. Setting $n = 0.95$, the average share of the labor force employed during the post-war period, and $x = 0.034$, this implies $f = 0.646$ on the balanced growth path. Second, the functional form for m implies

$$\bar{\mu} = f^{1-\eta} \mu^\eta,$$

⁴See <ftp://ftp.bls.gov/pub/special.requests/opt/mp/prod3.mfptablehis.zip>, Table 4. Between 1948 and 2007, productivity grew by 0.818 log points, or approximately 0.014 log points per year. The model assumes labor-augmenting technical progress, and so I must multiply \bar{s} by $1 - \alpha$ to obtain TFP growth.

⁵More precisely, I use the Bureau of Economic Analysis's Fixed Asset Table 1.1, line 1 to measure the current cost net stock of fixed assets and consumer durable goods. I use National Income and Product Accounts Table 1.1.5, line 1 to measure nominal Gross Domestic Product.

where $\mu \equiv m(v, u)/v$ is the number of hires per recruits. Using $f = 0.646$, $\mu = 8.33$, and $\eta = 0.5$, this second equation implies $\bar{\mu} = 2.32$. Note that this implies that the recruiter-unemployment ratio is $v/u = f/\mu \approx 0.078$. It follows that the share of recruiters in employment is $v/n \approx 0.004$, with 99.6 percent of employees devoted to production. Thus in this calibration, the implicit hiring costs are small, at least on average.

Finally, I consider the model both with $\sigma = 1$ and with higher values which allow for consumption-labor complementarity. It turns out that for values of σ in excess of 2.5, the depreciation rate implied by a capital-output ratio of 3.2 is negative, putting some discipline on this parameter. I set the parameter γ , governing the taste for leisure, to obtain a five percent unemployment rate along the balanced growth path; the exact value depends on the other calibrated parameters.

When I log-linearize around the balanced growth path, I confirm that the system is saddle-path stable, with two eigenvalues lying inside the unit circle and two lying outside. The stable eigenvalues, associated with the convergent dynamics, are 0.990 and 0.313. The low value of the second eigenvalue implies that when the state variables start away from eigenvector associated with the larger eigenvalue, they quickly converge to that eigenvector. I therefore assume in what follows that the economy starts on the eigenvector associated with the larger eigenvalue and ignore the short-lived transitional dynamics coming from the smallest eigenvalue.

In the frictionless model, I found that when $\sigma = 1$, a reduction in the capital stock raised employment and investment and reduced consumption. The solid blue in line Figure 3 confirms this result. Employment and investment increase sharply above trend, while consumption and labor productivity fall below trend during the transitional dynamics. In this case, the employment response is so strong that output is actually high when the capital stock is low. All of these effects quickly wear off, closing 1.7 percent of the gap to the steady state in every month. This is faster than the transitional dynamics in a model without an employment margin because the increase in employment accelerates convergence. For example, if the savings rate and employment were constant, the rate of convergence would be $(1 - \alpha)(g - \delta)/(1 + g) = 0.3$ percent per month.

The remaining lines in Figure 3 show various calibrations of the search model. The dashed red line shows the benchmark parameterization. The most noticeable impact of the search friction is to dampen the employment response. Intuitively, when employment is high, the efficiency of recruiters is low, discouraging the planner from doing too much recruiting. This offsets, but does not reverse, the incentive to raise employment. Because of the dampened employment response, output of the final good falls in the search model and the response of

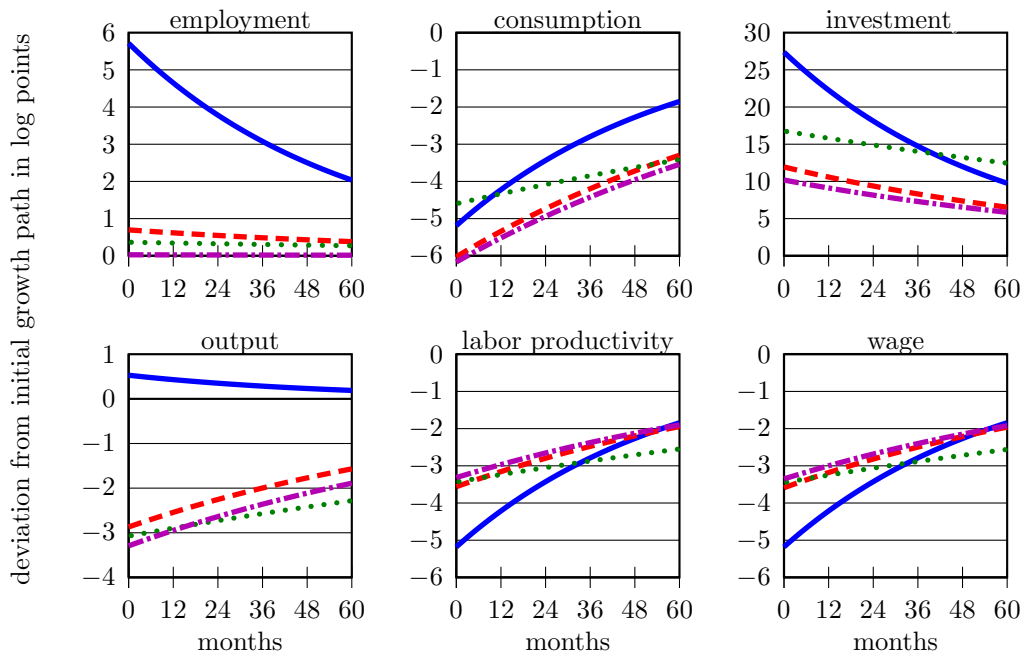


Figure 3: Transitional dynamics starting from ten percent below the trend capital stock, social planner's solution. The solid blue line shows the frictionless model with $\sigma = 1$. The dashed red line shows the benchmark parameterization of the search model. The dotted green line shows $\sigma = 2.5$, and the dash-dot purple line shows $\bar{\mu} = 0.8$. The remaining parameters are fixed at their benchmark values.

labor productivity is muted. Finally, search frictions prolong the transitional dynamics, but otherwise do not much affect the dynamics of consumption, investment, labor productivity (final output per worker), or wages.⁶

I consider several variations of the benchmark model. First, I raise the complementarity between consumption and labor supply to $\sigma = 2.5$, the highest value consistent with a nonnegative depreciation rate and a capital-output ratio of 3.2. The calibration target for unemployment implies $\gamma = 0.46$, and so employed workers optimally consume $1 + (\sigma - 1)\gamma = 1.69$ times as much as unemployed workers. This is far larger than the reduction in consumption at retirement would suggest is plausible (Aguiar and Hurst, 2005). Still, the model generates an increase in employment and investment following an adverse shock to the capital stock, with the response simply more muted than in the benchmark model.

I also consider the effect of the matching function on model outcomes. First I set $\bar{\mu} = 0.80$, which implies that each recruiter attracts only one worker per month. This nearly eliminates the response of employment, but has little effect on the other outcomes. I also examine the importance of the elasticity of the matching function. At one extreme, if $m(v, u) = \bar{\mu}u$, employment is fixed at $n = \bar{\mu}/(x + \bar{\mu})$. At the other, if $m(v, u) = \bar{\mu}v$, the transitional dynamics are identical to the ones in the frictionless model.⁷ Intuitively, diminishing returns to recruiting limits the willingness of the planner to increase recruiting in response to a shock that would otherwise encourage him to raise the level of employment. This issue disappears in the limit with $\eta = 1$. The bottom line is that no calibration of this model delivers a jobless recovery during the transition back to steady state.

3.2 Decentralization

As a prelude to introducing the rigid wage model, I discuss how the social planner's solution can be implemented through Nash bargaining over wages in a decentralized economy.

The problem of a household is unchanged from the frictionless model with rigid wages. In particular, a household with initial assets a_0 chooses a path for consumption $\{c_{e,t}, c_{u,t}\}$ to maximize the average utility of its members,

$$\sum_{t=0}^{\infty} \beta^t \left(n_t \frac{c_{e,t}^{1-\sigma} (1 + (\sigma - 1)\gamma)^\sigma}{1 - \sigma} + (1 - n_t) \frac{c_{u,t}^{1-\sigma}}{1 - \sigma} \right),$$

⁶I discuss the decentralization of the social planner's problem, and so wage determination, in the next section.

⁷This result holds if $\sigma = 1$, in which case a constant $\lambda_t A_t \kappa_t^\alpha$ solves equation (12), as in the frictionless first order condition (5); use this to eliminate κ from equation (4) and establish that the dynamics of marginal utility is identical in the two models. Even in this special case, the economy must start with initial conditions consistent with constant $\lambda_t A_t \kappa_t^\alpha$; otherwise the search model has some additional dynamics coming from the fact that employment is a state variable.

subject to a lifetime budget constraint

$$\sum_{t=0}^{\infty} q_0^t (n_t c_{e,t} + (1 - n_t) c_{u,t} - w_t n_t) = a_0,$$

taking as given a path for intertemporal prices q_0^t , the wage w_t , and the level of employment n_t . As in the frictionless model with rigid wages, the household problem yields the Euler equation (6), where λ_t is still the marginal utility of consumption for a hypothetical representative agent. This, combined with the household budget constraint, completely describes the household's behavior. I stress that, even in the decentralization of the planner's problem in the search model, the household cannot control its employment rate.

Next write the problem of a firm that starts with k_0 units of capital and n_0 employees. It chooses a time path for recruiting, employment, and capital to maximize

$$\sum_{t=0}^{\infty} q_0^t (A_t^{1-\alpha} k_t^\alpha (n_t - v_t)^{1-\alpha} + (1 - \delta)k_t - k_{t+1} - w_t n_t),$$

where $n_{t+1} = (1 - x)n_t + v_t \mu(\theta_t)$. The firm places v_t workers into recruiting, each of whom hires $\mu(\theta_t)$ new workers. The function μ is obtained from the matching function as $\mu(v/u) \equiv m(v, u)/v = m(1, u/v)$, decreasing in the recruiter-unemployment ratio. Note that recruiting is constant returns to scale at the firm level, although it generally has diminishing returns at the aggregate level. Each firm takes the aggregate recruiter-unemployment ratio $\theta = v/u$ as given when choosing how much to recruit, although the aggregate outcome will be determined in equilibrium.

To solve this problem, place a multiplier $q_0^t \xi_t$ on the law of motion for employment and take the first order conditions. The first order condition for capital yields

$$q_0^t = q_0^{t+1} (\alpha \kappa_{t+1}^{\alpha-1} + 1 - \delta),$$

where κ_t is again the capital-producer ratio. The first order condition for vacancies implies

$$(1 - \alpha) A_t \kappa_t^\alpha = \xi_t \mu(\theta_t).$$

And the first order condition for employment is

$$\xi_t q_0^t = q_0^{t+1} ((1 - \alpha) A_{t+1} \kappa_{t+1}^\alpha + \xi_{t+1} (1 - x) - w_{t+1}).$$

Eliminate the current and future multipliers from the first order condition for employment

using the first order condition for vacancies to get

$$(1 - \alpha)A_t\kappa_t^\alpha q_0^t = \mu(\theta_t)q_0^{t+1} \left((1 - \alpha)A_{t+1}\kappa_{t+1}^\alpha \left(1 + \frac{1 - x}{\mu(\theta_{t+1})} \right) - w_{t+1} \right).$$

Finally, use the household Euler equation (6) to eliminate the intertemporal prices from this equation and from the first order condition for capital to get equation (4) and

$$(1 - \alpha)A_t\kappa_t^\alpha \lambda_t = \beta \lambda_{t+1} \mu(\theta_t) \left((1 - \alpha)A_{t+1}\kappa_{t+1}^\alpha \left(1 + \frac{1 - x}{\mu(\theta_{t+1})} \right) - w_{t+1} \right). \quad (13)$$

Equation (13) differs from equation (12) because of the presence of the wage. Its logic is basically unchanged, however, still equating the value of a producer and a recruiter. The left hand side is the current marginal product of a producer evaluated at the current marginal utility of consumption. The right hand side is the discounted product of next period's marginal utility of consumption, the number of workers attracted by a recruiter, and the extra output net of wage costs from the newly hired workers and the workers who are freed up from recruiting next period.

To decentralize the planner's problem, I assume that workers bargain with employers to set wages, with an outcome is described by the Nash bargaining solution. A worker's threat point while bargaining is that he becomes unemployed, while a firm's threat point is that it loses the worker. Moreover, let $\phi \in (0, 1)$ denote the worker's bargaining power. Shimer (2010) proves that the wage satisfies

$$w_t = \phi(1 - \alpha)A_t\kappa_t^\alpha(1 + \theta_t) + (1 - \phi) \frac{\gamma\sigma c_t}{1 + (\sigma - 1)\gamma n_t}. \quad (14)$$

This is a weighted average of two terms. The first term is the marginal product of labor. This accounts both for the output the worker produces $(1 - \alpha)A_t\kappa_t^\alpha$, and for the fact that, if bargaining fails, the size of the firm shrinks and the firm must place additional workers into recruiting in order to maintain the same size the next period. The second term is the marginal rate of substitution between consumption and leisure. In the frictionless model, these two terms are equal and both are equal to the wage. With search frictions, the marginal product of labor generally exceeds the marginal rate of substitution and the wage lies between these two levels. At this wage, firms are happy to hire workers and households are happy to supply labor.

In general, the equilibrium does not decentralize the planner's problem. But suppose that $m(v, u) = \bar{\mu}v^\eta u^{1-\eta}$, so $\mu(\theta) = \bar{\mu}\theta^{\eta-1}$ for some $\eta \in [0, 1]$. If $\phi = 1 - \eta$, one can verify that equations (13) and (14) reduce to the planner's condition in equation (12), and so the

decentralized equilibrium is efficient. This is a generalization of the Mortensen (1982)–Hosios (1990) efficiency condition. Conversely, any other wage path that has a different expected present value leads to an inefficient equilibrium.⁸

3.3 Rigid Wage Model

The rigid wage model is identical to the decentralization above, except that the wage does not satisfy the Nash bargaining solution, but is instead constant at some level regardless of the sequence of shocks. That is, both the household’s problem and firm’s problem are unchanged, yielding equations (4) and (13).⁹ All that changes is the wage equation (14), which I replace with a simple assumption of a wage that grows with productivity, $w_t = (1 + g)^t w_0$. I verify that under appropriate conditions, firms do not want to fire their workers at that wage and workers do not want to quit their jobs.

Before proceeding to the business cycle analysis, some comparative statics are useful for understanding how the search model differs from the frictionless one. In the model without search frictions and a rigid wage, the balanced growth path was degenerate unless a borderline condition, $\beta R = (1 + g)^\sigma$, held. This is no longer true with search frictions. Instead, the balanced growth version of the Euler equation (4), pins down the capital-producer ratio κ :

$$1 = \beta(1 + g)^{-\sigma} (\alpha A \kappa^{\alpha-1} + 1 - \delta).$$

Then the balanced growth version of the first order condition for recruiting, equation (13), pins down the recruiter-unemployment ratio θ as a function of the wage per efficiency unit of labor:

$$\frac{w_t}{A_t} = (1 - \alpha) \kappa^\alpha \left(1 - \frac{\frac{(1+g)^{\sigma-1}}{\beta} + x - 1}{\mu(\theta)} \right).$$

Note that this condition ensures that the wage is always smaller than the full marginal product of labor, $(1 - \alpha) A_t \kappa^\alpha (1 + \theta)$. Thus firms are always willing to employ workers at the equilibrium wage. Next, the steady state equation for employment pins down n :

$$n = \frac{f(\theta)}{x + f(\theta)},$$

⁸Other wage paths may have the same expected present value but spread risks differently across workers and firms. For example, an employed worker may earn a constant wage while employed, although the wage depends on the labor market conditions when he is hired. This changes measured wages but does not alter the efficiency of equilibrium (Shimer, 2004).

⁹This is in contrast to the frictionless labor market model, in which the household problem changed in the presence of rigid wages.

where $f(\theta) \equiv m(\theta, 1)$ is the job finding probability for unemployed workers as a function of the recruiter-unemployment ratio. Using $\kappa = \frac{k}{A(n-\theta(1-n))}$, one then obtains the level of capital k . Finally, the balanced growth equation for capital pins down consumption:

$$c = (\kappa^{\alpha-1} - \delta - g) k.$$

The wage does not affect κ , but an increase in the wage raises $\mu(\theta)$ and so reduces θ . This in turn lowers employment. In general, the effect on capital and consumption is ambiguous, although it may be natural to focus on parameters for which an increase in recruiters raises the balanced growth number of producers $n - \theta(1 - n)$; this is always the case at the social optimum. But the important point to note is that there is a range of values for which the wage is consistent with a well-behaved balanced growth path.

It only remains to check whether the wage exceeds the marginal rate of substitution between consumption and leisure, so workers are willing to participate in the labor market:

$$w \geq \frac{\gamma\sigma c}{1 + (\sigma - 1)\gamma n}.$$

Unfortunately, I cannot proceed analytically. Instead, I ask what wages are consistent with this condition in the calibrated model. A wage of $w_t/A_t = 4.022$ decentralizes the social planner's solution. As w_t/A_t increases to 4.040, the recruiter-unemployment ratio falls to 0, driving the unemployment rate up to 1, and so at higher wages firms are unwilling to recruit workers. Conversely, as the wage declines lower, the recruiter-unemployment and employment rate rise. For values of the wage above $w_t/A_t = 2.995$, households are still willing to supply labor. But at still lower wages, the labor market would start to shut down as households refuse employment opportunities. But a large range of wages is consistent with workers willingly supplying labor to firms and firms willingly hiring workers.

Why does an equilibrium exist for such a large range of fixed wages? This is essentially the insight in Diamond (1971), extended to a business cycle analysis in Hall (2005). If all firms pay a wage w , no worker would accept a job at a wage much below w because doing so would be dominated by continued job search. On the other hand, if workers anticipate only getting jobs at a lower wage, they will be willing to do so. This strategic complementarity in the wage acceptance decisions is enough to leverage small search frictions into a large region of wage indeterminacy. Indeed, with appropriate assumptions about wage setting, the equilibrium of a search economy need not converge to the competitive equilibrium as the search frictions become arbitrarily small.

The balanced growth analysis illustrates another point: although a balanced growth path exists for a range of fixed wages, the employment rate is extremely sensitive to the wage.

A 0.1 percent increase in the wage, from 4.022 to 4.026 raises the unemployment rate from 5 percent to 6.3 percent, while a 0.5 percent increase in the wage shuts down the economy entirely. This occurs because the increase in the wage reduces the profitability of recruiting a worker at a fixed marginal product of labor. With less recruiting, employment falls as well.

To study the model out of the balanced growth path, I use the same parameter values as in the social planner’s problem, with the wage set at a level such that the equilibrium of the deterministic economy is socially optimal. I start by computing the eigenvalues of the linearized system of equations to verify that it is saddle-path stable. The two stable eigenvalues are 0.998 and 0.291 in the benchmark model. The low value of the smallest eigenvalue ensures that the system quickly converges to the eigenvector associated with the larger eigenvalue. And the increase in the larger eigenvalue—it was 0.990 in the flexible wage model—indicates that the transitional dynamics are substantially slower in the rigid wage model.

I find that in the rigid wage model, a reduction in the capital stock lowers output, employment, consumption, and investment for a broad range of parameterizations of the model. Moreover, the declines are roughly equal in magnitude. Figure 4 illustrates this both in the benchmark model and in several variants of it. The solid blue line shows the frictionless model, identical to the model where the elasticity of the matching function is $\eta = 1$. This delivers the exact theoretical result that employment, consumption, investment, and output all decline by an equal magnitude.¹⁰ The dashed red line shows the benchmark model, where the decline in employment, output, and investment is slightly smaller. The dotted green line shows a higher value of the complementarity parameter, $\sigma = 2.5$; this scarcely affects the results. The dash-dot purple line shows a much more frictional economy, $\bar{\mu} = 0.8$, so recruiters contact one worker per month. Curiously in this case it is possible to generate an increase in investment following the initial shock, and hence a rather strong recovery. This does not look like a jobless recovery.

I stress that all the figures except for employment are expressed relative to trend; there is no trend in employment. The figures therefore suggest a jobless recovery, with anemic growth in employment accompanying trend growth (at below trend levels) in consumption, investment, and output.

There is one interesting difference between the rigid wage model with a frictionless labor market and the rigid wage model with search frictions: in the first model, the economy never recovers from the adverse shock, while in the second model it does if $\eta < 1$. The reason for the recovery is instructive. Following an adverse shock to the capital stock, suppose

¹⁰This result holds in the search model with $\eta = 1$ for arbitrary values of the other parameters.

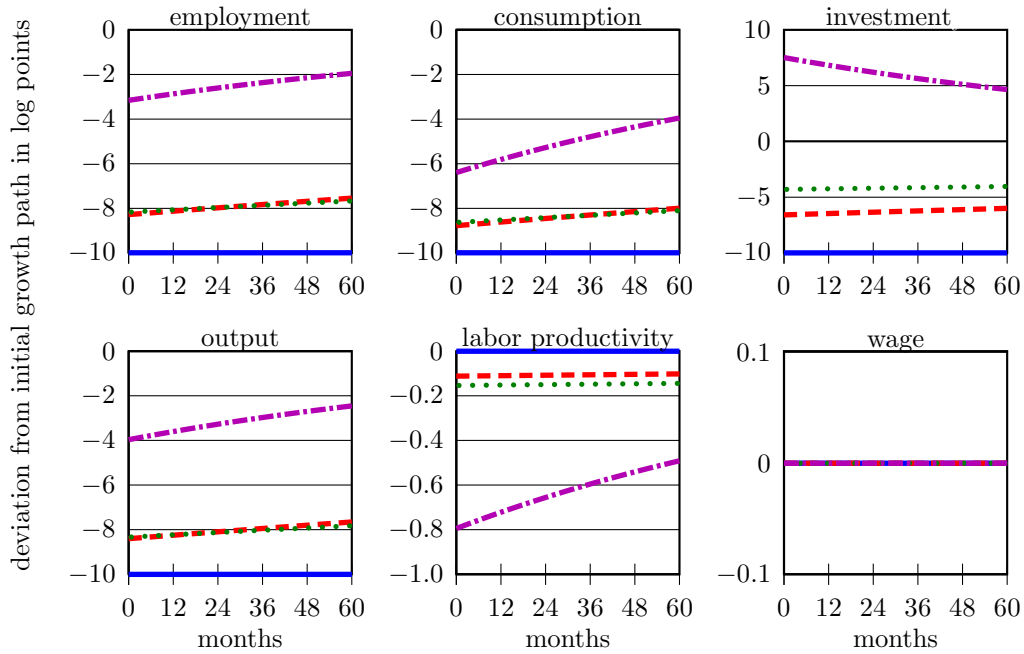


Figure 4: Transitional dynamics starting from ten percent below the trend capital stock, rigid wage model. The solid blue line shows the frictionless model with $\sigma = 1$. The dashed red line shows the benchmark parameterization of the search model. The dotted green line shows $\sigma = 2.5$, and the dash-dot purple line shows $\bar{\mu} = 0.8$. The remaining parameters are fixed at their benchmark values.

employment fell by the same amount. This pushes down the recruiter-unemployment ratio, allowing firms to economize on recruiting. In other words, output of the final good does not fall by as much as the decline in capital and employment. It is then possible to invest a bit more, leading the economy to eventually return to the balanced growth path. Indeed, if $\mu(\theta)$ is constant, as is the case when matching depends only on the number of recruiters, one can prove that this channel is shut down. An adverse shock to the capital stock is permanent, leading to an equal permanent decline in employment, consumption, investment, and output relative to trend, as in the model without search frictions.

Finally, note that in a flexible wage economy, the wage would fall after this adverse shock. It follows that the rigid wage is always higher than the marginal rate of substitution between consumption and leisure. Put differently, when the unemployment rate is high because wages are too high, individuals are happy to work when they have an opportunity. In addition, the wage is less than the marginal product of labor as long as the shock is not so large that firms stop recruiting new workers. Thus the model can handle a sharp slowdown in recruiting following an adverse reallocation shock without necessitating any change in the wage.

4 Conclusion

This paper studied the transitional dynamics in an economy that has lost some of its capital stock. If the wage does not fall from its trend, I showed that the model can explain a persistent decline in output, employment, consumption, and investment. This is consistent with individually rational behavior in a search model: firms are still willing to hire workers at the high wage, but they simply cut back on recruiting; and workers are happy when they are able to get a job at this high wage. The conclusion is robust to the specification of preferences and the nature of the matching technology, arguably the most controversial ingredients in the theory.

The paper has deliberately tried to avoid a discussion of the nature of the initial adverse shock. One of the points is that, for the purpose of exploring the recovery from that shock, the nature of the shock (and preferences, and search technologies) is unimportant as long as wages are rigid. Still, the fact that the model is able to deliver a protracted contraction in response to a decline in the capital stock suggests that many different shocks may have a similar effect on economic outcomes. For example, imagine firms producing heterogeneous goods. The final good, used both for consumption and investment, is an aggregate of the intermediate goods. When final goods are invested, the capital becomes specific to a particular variety of intermediate good. Finally, idiosyncratic shocks change the composition of intermediate goods used to produce the final good. When a shock reduces the demand for

a particular variety, effectively the associated capital becomes worthless. In a flexible wage model, workers would immediately be reallocated to more productive uses. But with rigid wages this is impossible and the reallocational shock may lead to a protracted contraction.

I have ignored the government sector in this paper. One interesting question is what impact countercyclical government spending has in this type of model. It is straightforward to introduce a government financed by lump-sum taxes. In particular, with lump-sum taxes, Ricardian equivalence holds and so the timing of taxation is irrelevant. In a flexible wage model, wasteful government consumption, or government consumption that enters additively into individuals' utility functions, raises labor supply by making households feel poorer. This carries over to the search model as well. An increase in government consumption lowers wages and labor productivity and raises output, though by less than the increase in spending. With rigid wages, a temporary increase in government consumption crowds out private investment. The decline in the capital stock then reduces employment, consumption, and output.¹¹ Thus this model, despite its arguably Keynesian rigid wage and noncompetitive labor market, predicts that countercyclical fiscal expansions will have perverse and unintended consequences. On the other hand, policies designed to reduce employment costs, such as payroll tax cuts, may have significant, long-lasting, and salutary effects in a depressed economy.

References

- Aguiar, Mark, and Erik Hurst, 2005. "Consumption versus Expenditure." *Journal of Political Economy*. 113 (5): 919–948.
- Barro, Robert J., 1977. "Long-Term Contracting, Sticky Prices, and Monetary Policy." *Journal of Monetary Economics*. 3 (3): 305–316.
- Diamond, Peter A., 1971. "A Model of Price Adjustment." *Journal of Economic Theory*. 3 (2): 156–168.
- Elsby, Michael, Bart Hobijn, and Ayşegül Şahin, forthcoming. "The Labor Market in the Great Recession." *Brookings Papers on Economic Activity*.
- Hagedorn, Marcus, and Iouri Manovskii, 2008. "The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited." *American Economic Review*. 98 (4): 1692–1706.

¹¹In contrast, an increase in government investment, if more productive than private sector investment, may be expansionary. Still, the model does not explain why this investment should only be performed during recessions.

- Hall, Robert E., 2005. "Employment Fluctuations with Equilibrium Wage Stickiness." *American Economic Review*. 95 (1): 50–65.
- Hansen, Gary D., 1985. "Indivisible Labor and the Business Cycle." *Journal of Monetary Economics*. 16 (3): 309–327.
- Hosios, Arthur J., 1990. "On the Efficiency of Matching and Related Models of Search and Unemployment." *Review of Economic Studies*. 57 (2): 279–98.
- Merz, Monika, 1995. "Search in the Labor Market and the Real Business Cycle." *Journal of Monetary Economics*. 36 (2): 269–300.
- Mortensen, Dale T., 1982. "Property Rights and Efficiency in Mating, Racing, and Related Games." *American Economic Review*. 72 (5): 968–979.
- Petrongolo, Barbara, and Christopher A. Pissarides, 2001. "Looking into the Black Box: A Survey of the Matching Function." *Journal of Economic Literature*. 39 (2): 390–431.
- Pissarides, Christopher A., 1985. "Short-Run Equilibrium Dynamics of Unemployment, Vacancies, and Real Wages." *American Economic Review*. 75 (4): 676–690.
- Rogerson, Richard, and Robert Shimer, forthcoming. "Search in Macroeconomic Models of the Labor Market." in Orley Ashenfelter, and David Card (ed.), *Handbook of Labor Economics*, vol. 4. Elsevier.
- Shimer, Robert, 2004. "The Consequences of Rigid Wages in Search Models." *Journal of the European Economic Association*. 2 (2–3): 469–479.
- Shimer, Robert, 2005. "The Cyclical Behavior of Equilibrium Unemployment and Vacancies." *American Economic Review*. 95 (1): 25–49.
- Shimer, Robert, 2007. "Reassessing the Ins and Outs of Unemployment." Mimeo.
- Shimer, Robert, 2010. *Labor Markets and Business Cycles*, Princeton University Press.
- Silva, José, and Manuel Toledo, 2009. "Labor Turnover Costs and the Cyclical Behavior of Vacancies and Unemployment." *Macroeconomic Dynamics*. 13 (S1): 76–96.